DEPARTMENT OF MATHEMATICS

S4 MATHEMATICS INTERNAL SEMINAR FOR

30th JUNE 2019

O'LEVEL MATHEMATICSPAPER I & II
DETAILED DISTRIBUTION OF TOPICS

PAPER 1	PAPER 2
• Statistics • Matrices • Transformations • Linear Programming • Quadratic graphs • Inequalities • Construction • Bearings • Trigonometry • Probability • Operations • Fractions • Factorisation • Equations & Formulae • Circle Properties	• Set theory • Vectors • Business Math and Taxation • Kinematic graphs • 3 Dimensions • Functions • Coordinate Geometry • Ratios • Proportions • Decimals • Surds • Indices • Logarithms

A. MATRICES

1. If $A = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 \\ 5 & -3 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$, and $M = 2AB - C^2$

Find (i) M (ii) M^{-1}

2. Find the values of x and y from the matrix equation.

$$\begin{pmatrix} y & 4 & 2 \\ 3 & 1 & x \end{pmatrix} \begin{pmatrix} 3 \\ -4 \\ x \end{pmatrix} = \begin{pmatrix} 4 \\ x+17 \end{pmatrix}$$

Katorogo Food store shop sells Beans, Rice and Ground nuts at the following prices:

Beans (Ushs/kg)	Rice (Ushs/kg)	Ground nuts (Ushs/kg)
3500	4000	5000

On a certain weekend from Friday, Saturday up to Sunday, the following quantities were purchased from his shop.

Item	Friday	Saturday	Sunday
Beans (kg)	25	20	21
Rice (kg)	15	10	14
Ground nuts (kg)	12	6	3

(a) (i) Form a 3 X 1 cost matrix for the prices of the produce.
(ii) Form a 3 X 3 matrix to show the commodities Katorogo sold.
(iii) By matrix multiplication, find Katorogo's total earnings for the weekend.

(b) On which day did Katorogo get:
(i) The lowest sales revenue
(ii) The highest sales revenue
(iii) The difference of the above sales revenues

B. INDICES, SURDS AND LOGARITHMS

1. Solve for x: (i) $49^x + 7^{2x} = 686$ (ii) $\log(3x+2) - 1 = \log(x-4)$

2. Use logarithm tables to evaluate $\sqrt{\dfrac{(0.4532)^2 \text{x } 0.8925}{1.704}}$.

3. Given that $\frac{3}{3+\sqrt{5}} + \frac{3\sqrt{5}}{3-\sqrt{5}} = P + Q\sqrt{5}$. Find the values of P and Q.

C. QUADRATICS

1. (a) Copy and complete the table below for the function.

$y = 9 + 3x - 2x^2$

x	−3	−2	−1	0	1	2	3	4
$9+3x$		3		9	1		1	1
$-2x^2$				0	−2		1	
y				9	1	7	0	

(b) Use the completed table to draw the graph of $y = 9+3x-2x^2$ in the range $-3 \leq x \leq 4$.

(c) Use the graph to determine the roots of;
(i) $9 + 3x - 2x^2 = 0$
(ii) $14 + x - 2x^2 = 0$

(d) State the maximum value of the function.

2. Draw the curve $y = 2x^2 + 5x - 3$ for $-4 \leq x \leq 2$ and using a scale of $2\,cm$ to $1\,unit$ along the $x-$axis and $1\,cm$ to $1\,unit$ along the $y-$axis.

(a) State: i) the minimum value of the function.

ii) the range of values of x for which $2x^2 + 5x - 3 < -1$.

(b) Use your graph to solve the equation $2x^2 + 4x = 0$.

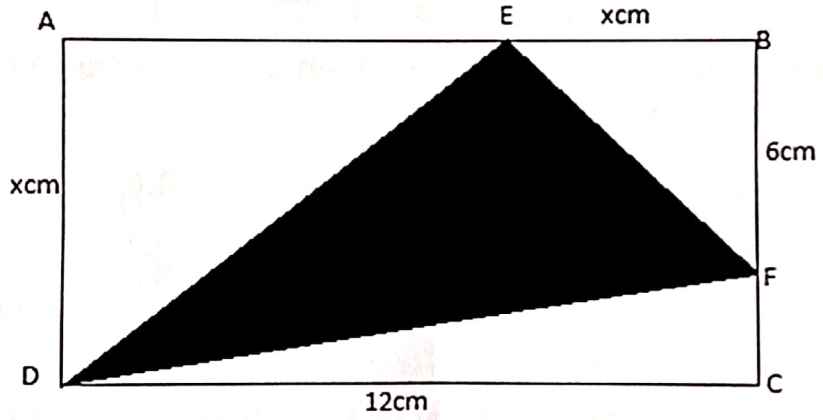
3. Given the matrix T = $\begin{pmatrix} 2x & 3 \\ x & x \end{pmatrix}$. Find the values of x for which det. T = 2.

4. Simplify the expression $\frac{2x^2 - 5xy - 3y^2}{(x - 3y)}$.

5. Given that $\begin{pmatrix} 3a & a-8 \\ -6 & a-2 \end{pmatrix}$ is a singular matrix; find the values of a.

6. Given that $212_{\mathbf{n}} = 25_{\mathbf{nine}}$, find the base that **n** represents

7. In the figure below, ABCD is a rectangle 12cm long and x cm wide. Given that $\overline{EB}$ = xcm, $\overline{BF}$ = 6cm and that the unshaded area is 52cm^2,



find; (a) The value of x b) The shaded area.

D.SET THEORY AND LOGIC

1. In a group of 50 foreign tourists who visited mountain Rwenzori, 28 speak French (F), 20 speak Spanish (S) and 18 speak German (G). 12 tourists speak both French and Spanish. 9 speak both German and French, 5 speak Spanish and German only.
The number of tourists who speak all the three languages equals the number of those who do not speak any of these languages.

(a) Determine;
(i) The number of tourists who speak all the three languages
(ii) The number of those who speak at most one language.
(b) If a tourist is selected at random from the group, find the probability that he or she speaks;

(i) At least two languages
(ii) None of the languages.

E. STATISTICS

1. The time taken for each bulb to burn out in a sample of 50 bulbs on a city street was recorded as in the table below.

Time(hours)	8-10	11-13	14-16	17-19	20-22	23-25
No. of bulbs	4	7	y	15	8	5

(a) (i) Find the value of y. (ii) Determine the mean.
(ii)Construct a frequency distribution table.
(b) On the same diagram, Draw
(i) A histogram and use it to estimate the modal time.
(ii) A frequency polygon.
(c) Draw a cumulative frequency curve and use it to estimate the
(i) median
(ii) The number of bulbs with burning time between 25 and 12 and 18 hours.

2. Forty students carried out an experiment and recorded the following measurements.

4.7	2.7	2.3	4.6	3.7	2.8	2.9	3.6
4.9	3.9	4.5	3.4	4.2	3.5	1.7	1.1
2.0	3.7	3.3	3.8	3.8	1.8	3.1	3.6
3.1	1.4	1.6	2.1	2.8	2.6	3.3	4.0
3.2	4.3	3.5	2.4	4.4	4.1	2.9	3.2

(a) Draw a frequency distribution table starting with 1.0 – 1.4.

(b) Hence state the: i) class interval, (ii) modal class.

(c) Calculate the mean and median of the data.

d) Draw a histogram and use it to estimate the mode.

F. PROBABILITY

1. A bag contains 3 green and 2 red balls. Two balls are randomly selected from the bag without replacement. Find the probability that: (i). Both are of same color. (ii). The second ball is red.

(iii). They are of different colors.

2. A regular octahedron, whose faces are numbered 1 to 8 is thrown together with a coin. Show the possible outcomes in a table. Find the probability of getting;

(i). "T" with prime number, (ii). "H" and a number less than 4.

G.COMMISSION, INTEREST AND HIRE PURCHASE

1. A saleswoman earns a basic salary of sh. 10,000 per month. In addition she is also paid a commission of 5% for sales above sh.15, 000. In a certain month she sold goods worth sh. 120,000 at a discount of 2%. Calculate her total earning that month.

2. A certain bank charges compound interest on money borrowed. A businessman borrowed sh. 16,000 from the bank. He paid back sh. 25,000 after two years. Find the interest rate per annum.

3. A Galaxy smart phone was bought on hire purchase. A deposit of 50,000 was paid and a 15 monthly installments of sh. 15,000 was required.
   (a) Calculate the total amount paid on hire purchase.
   (b) If the hire purchase price is 20% higher than the cash price, find the cash price.

H. FUNCTIONS

1. Given that $h(x) = \frac{x-1}{4x+8}$. Find (i) $h^{-1}(x)$ (ii) $h^{-1}(2)$

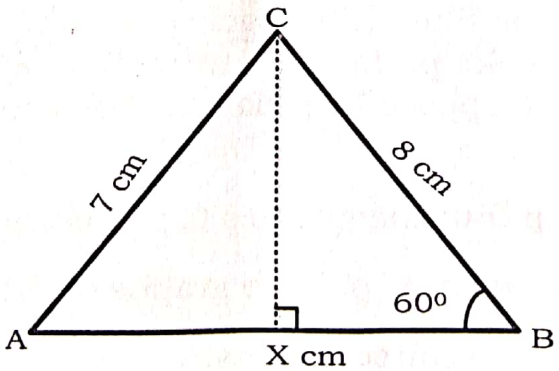
   (iii) The value of x for which $h(x)$is meaningless.

2. Given that $f(x) = 2x + 1$ and $g(x) = x - 5$. Find:

   (i) $f(-2)$ (ii) $gf(x)$ (iii) $fg(x)$
   (iv) the value of t, if $f(2t) = g(t) - 3$

I. TRIGONOMETRY

1. The figure below shows triangle ABC where AB = x cm, BC = 8cm, AC = 7cm and $\angle ABC = 60^o$. Calculate the area of triangle, correct to 2 significant figures.

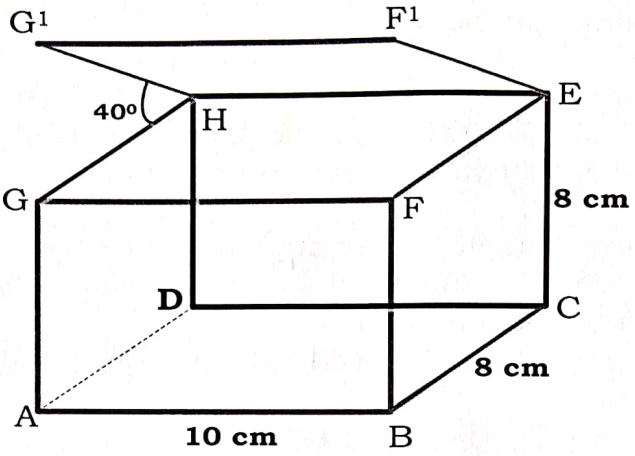


2. The angles of elevation of the highest point of a building from two points S

and T are **60°** and **30°**respectively. If T is vertically above S and ST = 10m. Calculate the height of the building above the level of S.

J. THREE DIMENSIONS (3D SHAPES)

1. The figure below shows a box ABCDEFGH with open lid G^1F^1EH.



Use the figure to calculate;

(a) (i) the length of line BD (ii)the height F^1 above FE.

(iii) the length of projection of AE onto plane ABCD,

(b) The angle between (i) GF and line AC (ii) DF and the plane ABCD

(iii) BH and the plane CDHE.

(c) The angle between the planes;

(i) ABEH and GFEH

(ii) GFDC and ABCD

K. LINEAR PROGRAMMING AND INEQUALITIES

1. Mr. Kato is going to bake chocolate cakes and yellow cakes to sell. He wants at least two chocolate cakes. Besides, he wants more yellow cakes than chocolate cakes. Due to limited time and facilities, he cannot bake more than ten cakes. The chocolate cakes are to be sold for Shs. 1500 each and the yellow cakes for Shs. 1000 each. To make profit, more than Shs. 8000 must be realized from the sales. (suppose he bakes x chocolate cakes and y yellow cakes).

a) Write down **four** inequalities to represent this information.

b)(i) On the same axes, plot the graphs of the inequalities and shade the un wanted regions.

(ii) List all the possible numbers of chocolate cakes and yellow cakes Mr. Kato can bake.

c) How many cakes of each type should Mr. Kato bake in order to make the maximum profits?

2(a) Express $19 < 3(x+2) < 35$ in the form $a \leq x \leq b$.

(b) Solve the inequality $x^2 + 3x - 10 < 0$.

L. BUSINESS MATHS

1. Morgan is a tourist to Uganda for his first time. He has US$ 1,200 which he changes to Uganda shillings (Ug. Shs) at a rate of $1 = Ug. Shs 3,000. If he has a balance of Ug. Shs 900,000 after all expenses find (a) the amount of money spent in Ug. Shs.

   (b) his balance in US dollars.

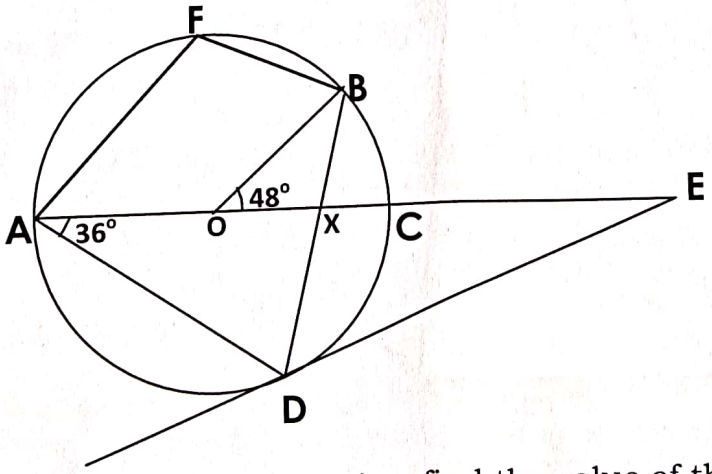
2. The income tax rates of a certain country are as follows:

Income (Ug shs)	Tax rates
0 - 120,000	0
120,001 – 400,000	10
400,001 – 800,000	20
800,001 and above	40

Determine the taxable income of an employee who paid Shs 92,000/= as income tax.

M. CIRCLE PROPERTIES

1. In the figure below, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. A, O, X and C are points on a straight line. DE is a tangent to the circle at D. Angle BOC= 48^{0}and angle CAD = 36^0.



(a) Giving reasons or otherwise, find the value of the following angles:-

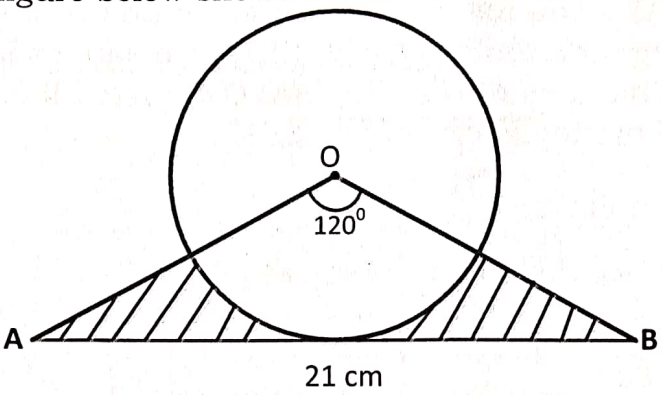
(i) Angle CBA

(ii) Angle AFB

(iii) Angle BDE
(iv) Angle CED
(v) Angle ADB

(b) It is also given that AX = 12 cm, XC = 4 cm, DB = 14 cm and DE = 15 cm. Calculate: (i) DX (ii) AE

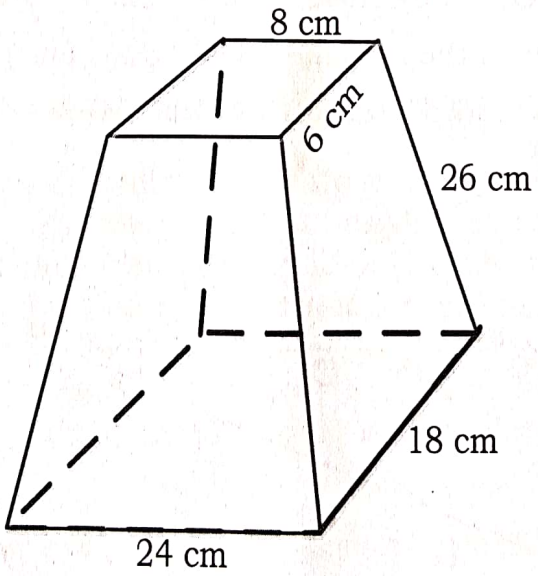
2. The figure below shows a circle centre O.



The line AB=21cm is a tangent to the circle such that **OA=OB** and angle **AOB=120⁰.** Calculate the area of the shaded region.

N. MENSURATION

1. The figure below shows a frustum with a rectangular base which measures 18cm by 24cm and the top measuring 6cm by 8cm. The slant edges are 26cm long each.



(a) Find the height of the original pyramid.
(b) Calculate the volume of the frustum.

O. TRANSFORMATIONS

1. Find the values of x *and* y for which $\boldsymbol{P}'(9,2)$ is the image of $\boldsymbol{P}(x,y)$ under transformation matrix $\boldsymbol{M} = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$ followed by $\boldsymbol{N} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$.

2. Identify the matrix of transformation that maps the unit square ***OIKJ*** onto $O(0,0), I'(2,7), K'(3,7), J'(1,0)$.

3. A triangle with vertices **A** (2, 0) , **B** (1, -3) and **C** (-2, 1) undergoes two successive transformations **T** followed by **R** to give the image **A"** (18, -2) **B"** (0, -4) and **C"** (-15, 3). Given that $\mathbf{T} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$.

(a) Find (i) the coordinates of triangle $A'B'C'$.

(ii) transformation matrix **R**. (iii) matrix **T** followed by **R**.

(b) Given that the area of triangle **ABC** is $6\frac{1}{2}$ square units, use the matrix obtained in b(iii) to determine the area of triangle $A"B"C"$.

P. CONSTRUCTION

1. Using a ruler and a pair of compasses only , construct in a single diagram.

(a) Quadrilateral OPQR in which line PQ = 6cm line QR = 4cm, line RO = 7cm, line OP = 8cm and angle PQR = 135^0.

(b)(i) A circle which touches line OP at point P and passes through point Q.

(ii) Measure the radius of the circle and hence calculate the area of the minor segment above PQ.

Q. SETS AND LOGIC

1. If $n(\mathcal{E}) = 17$, $n(P \cup Q) = 12$, $n(Q') = 8$, $n(P) = 10$, find (i) $n(P \cap Q)$ (ii) $n(P' \cap Q')$.

2. A total of **100** vehicles were inspected and **60** vehicles passed the road worthy test. The rest of the vehicles (remainder) had faults in: Brakes (**B**), Lights (**L**) and steering (**S**) as follows;- $n(B \cap L \cap S) = 3$; $n(B \cap S^I \cap L^I) = 12$; $n(B \cap S) = 5$; $n(B \cap L) = 8$; $n(S \cap L \cap B^I) = 2$ and $n(S \cap L^I \cap B^I) = n(L \cap B^I \cap B^I)$.

(a) Represent the given information on a Venn-diagram.
(b) How many vehicles had (i) Faulty steering.

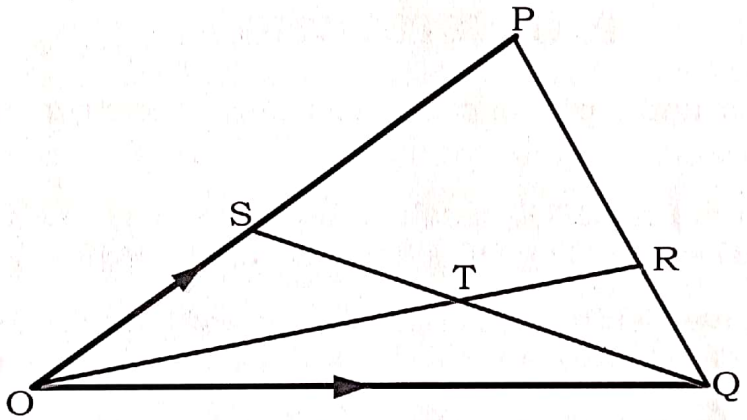
(ii) the fault only.

(c) If a vehicle is chosen at random; find the probability that it had at least **two** faults.

R. VECTORS

1. Given the column vectors $\boldsymbol{a} = \begin{pmatrix}1\\2\end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix}-3\\-6\end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix}2\\-3\end{pmatrix}$ and that $\boldsymbol{p} = 3\boldsymbol{a} - \frac{1}{3}\boldsymbol{b} + \boldsymbol{c}$, express $\boldsymbol{p}$ as a column vector. Hence, $|\boldsymbol{p}|$.

2. In the figure OPQ is a triangle in which **OS** = $\frac{3}{4}$**OP** and PR: RQ = 2:1, Lines OR and SQ meet at T.



(a) Given that **OP** = **p** and **OQ**=**q**. Express the following vectors in terms of **p** and **q**; (i) **PQ** (ii) **OR** (iii) **SQ**
(b) Given that **ST**=*h***SQ** and **OT** = *k***OR**. Determine the values of h and k.
(c) Hence, show that S, T and Q are collinear.

S. RATIOS

1. Given that $x:y=7:4$ and $y:z=5:6$, find the ratio $x:y:z$.

2. Give that $\frac{3x-4y}{3x+2y}=\frac{1}{2}$, find the ratio $x:y$.

T. VARIATIONS

1. If p varies jointly as q and r squared, and p = 225 when q = 4 and r = 3, find p when q = 6 and r = 8.

2. A quantity R varies partly as the square of V and partly as the cube of V. When V = 20, R = 416 and when V = 40, R = 3264.

(a) Form an equation relating R and V.

(b) Determine the value of R when V = 30.

U. TRAVEL GRAPHS

1. John starts cycling from his home at exactly 8:00 am at a steady speed of 16 kmh^{-1} to attend a meeting at the Gombolola headquarters, 20 km away. At the same time, Opio is sent from the Gombolola headquarters and comes cycling at a steady speed of 6 kmh^{-1} to inform John that the meeting was postponed.

   Unfortunately, at exactly 8:30 am, John gets a puncture and works on it for 20 minutes before proceeding his journey carefully at 8 kmh^{-1}.

   Using a scale of 1 cm to 1 km on vertical axis and 1 cm to 10 minutes on horizontal axis;

   (a) Draw the distance-time graphs on the same axes for both John and Opio.

   (b) Use your graphs to determine;

   (i) the time John met Opio.

   (ii) the time when they were 3 km apart.

   (iii) The distance from the point where they met to the Gombolola headquarters.

2. Two taxis **A** and **B** move off from rest in the same direction on a straight road. The speed of taxi **A** increases at a uniform rate of **2m/s** while taxi **B** moves as shown in the table below:

Time (s)	0	1	2	3	4	5	6	7	8
Speed (m/s)	0	0.5	1.5	4	10	15	18	19.5	20

(a) Draw on the same axes the speed-time graphs of taxis **A** and **b** using the scale of **1cm** to represent **1** second on the x-axis and **1cm** to represent **2m/s** on the y-axis.

(b) Using the graphs in (a) above, find the;-

i. Time and speed when taxi **B** overtook taxi **A**.

ii. Difference in the speed of the vehicles after **6** seconds.

(c) Distance covered by taxi **A**.

V. COORDINATE GEOMETRY

1. Find the equation of a line passing through origin and the mid-point of line a line joining two points A(1, 3) and B(5, -9). Find also the length of the line AB.
2. A straight line passing through the points P(5, k) and Q(k,6) is perpendicular to the line whose equation is 4y-3x=12. Find the value of k.
3. A line $y = 2x - 6$ cuts the x-axis at point M and y-axis at point N. Determine the coordinates of points M and N.

END